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DYNAMIC INTERACTIONS BETWEEN PREY AND PREDATOR WITH COOPERATION AND ALLEE EFFECT: DETERMINISTIC AND STOCHASTIC APPROACH

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In this paper, we investigate the behavior of a predator–prey model with cooperation and Allee effect considering both deterministic and stochastic approaches. The main aim of this paper is to investigate the effect of environmental fluctuation in a deterministic predator–prey model. During the analysis of the deterministic model, it is shown that the system has saddle-node point of co-dimension 1, Hopf bifurcation and Bogdanov–Takens bifurcation of co-dimension 2. To study the effect of environmental fluctuation, we use perturbation to the birth rate of prey and death rate of predator density by Gaussian white noise. The persistence of the model and the stationary distribution is shown by forming a suitable Lyapunov function. Finally, numerical simulations are performed to validate the theoretical findings.

Keywords: Predator–Prey Model; Cooperation; Allee Effect; Stochastic; Hopf Bifurcation; Bogdanov–Takens Bifurcation.

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Effect of Environmental Fluctuation in the Dynamics of a Three-Species Food Chain Model with Sexually Reproductive Generalized Type Top Predator and Crowley-Martin Type Functional Response Between Predators

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Abstract
The main aim of this paper is to explore the dynamical behavior of a three-species prey-predator interaction model with sexually reproductive generalized type top predator under the consideration of environmental fluctuations. Our discussion has involved a continuous tritrophic food chain model with Crowley-Martin senses and Holling type II functional responses. For the deterministic model, the existence of equilibria as well as boundedness of the solution has been established here. The feasibility and local stability of the interior and non-interior equilibrium points has been investigated. The global dynamics of the co-existence of all the three species are shown at the interior equilibrium point. To determine the direction of Hopf bifurcations under the non-hyperbolic case, the first Lyapunov number is computed using the center manifold theorem. Several bifurcation analyses are performed at the interior equilibrium point. The effect of environmental fluctuation on some of model parameters are studied here through the verification of existence of unique positive global solution, existence and persistence of stationary solution. Numerical simulation has been carried out to illustrate the theoretical findings by using the MATLAB and Maple software packages and finally some concluding remarks are given.

Keywords Holling type II functional response · Crowley-Martin functional response · Global stability · Hopf bifurcation · Center manifold theory · Stochastic model · Persistence in mean

1 Introduction

The ecological world is a complex interconnected system of species interacting with one another. Competition can take many forms, but food competition (intra-species) and prey-predator relations (inter-species) are the most common [1, 2]. Several studies have been accomplished on prey-predator interaction using Lotka-Volterra type functional response [3]. Typically, the prey-predator models combine functional responses, which describe how each predator responds to changes in prey density during a given period of time [4–6]. According to Holling, four types of functional responses were developed as a result of simplification of prey searching, prey consumption, and environmental complexity, (referred to as Holling types I, II, III, and IV) on the basis of empirical field data [7–11]. The nature of prey consumption rate when a predator consumes a population of prey determines the form of the functional response (linear, hyperbolic, and sigmoidal). A number of studies have been conducted that consider Holling type II functional responses in prey-predator systems [12].

Traditionally, prey-predator models assume that functional response is entirely dependent on prey density, which has come under increased scrutiny of late. As suggested by Arditi and Ginzburg [13], another prey-predator model assumes that functional responses are affected by the densities of both prey and predator populations. According to them, the traditional functional response is based on the

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